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Optimal Impulsive Control of Relative Satellite Motion

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The periodic orbit/Floquet theory solution to relative satellite motion is extended to include optimal two-impulse stationkeeping about a specified natural solution. Simulations of this method are performed in a model including the entire geopotential through order 14, as well as air drag. For a cluster roughly 4 km across at an orbital altitude of 637 km, and at an inclination of about 57 deg, closed-loop maneuver requirements of <1 cm/s per day are achievable for up to one week. Departure of the numerical integration from the periodic orbit/Floquet theory model causes maneuver requirements to increase near the end of the simulation time. Most of the maneuvering is performed to maintain in-track spacing, and a better treatment of the in-track dynamics may lower maneuver requirements further.

Introduction

IN the past few years, there has been considerable interest in the dynamics and control of satellite clusters and satellite formation flying. Here, these efforts are separated into two broad groups. In the first approach, the cluster is restricted to a special geometry, or it is presumed that the cluster will be frequently rearranged. The prototype for this type of cluster operation is the only satellite formation flying that is currently actually performed: rendezvous of a manned spacecraft with another object in orbit. In this case, the total maneuver budget for rendezvous is typically well over 10 m/s.

However, this represents fuel expenditure at a level well above what can be supported for even a few weeks and certainly not for years. For comparison, stationkeeping costs for geosynchronous satellites are on the order of 10m/s per year, which is a far more feasible level of maneuvering. Now, to achieve cluster formation flying at such a small maneuvering level, it is obvious that the trajectories of individual satellites must be natural, free flying trajectories most of the time. To make this possible, it is essential that the dynamics of formation flight be considerably improved over the Clohessy–Wiltshire[1] dynamics model, which includes only the linearization of two-body dynamics about a circular orbit. Improving the dynamics solution will reduce the amount of spurious maneuvering to cancel predictable, stable perturbations.

In Ref. 2, a new solution for satellite relative motion was introduced, which included in the dynamics model all zonal harmonics. The solution was based on a nearly circular periodic orbit, with Floquet theory used to obtain the first-order relative motion solution. In the inertial frame, the solution for the inertial state vector $\mathcal{I}(t)$ can be written as

$$\mathcal{I}(t) = R_Z^{(2)T}\left\{\mathcal{N}_0(t) + \left(\mathcal{R}^{(2)}\right)^T F(t)e^{Jt}z(t_0)\right\} \tag{1}$$

The regression of the nodes (the preceding z axis rotation matrix $\mathcal{R}_Z^{(2)}$) appears naturally as a rotation of the inertial reference frame necessary to get the orbit to be periodic. The periodic orbit in this regressing frame (the nodal frame) is written as $\mathcal{N}_0(t)$ and is easily expressed as a Fourier series. The rotation matrix $\mathcal{R}^{(2)}$ transforms from the usual orbital reference frame, for example, the radial, orbit normal, and in-track directions, to the nodal frame and is periodic. The matrix $F(t)$ is the periodic Floquet modal vector matrix, and this can also be easily reduced to a Fourier series with the same period as the orbit. The matrix J is the constant matrix of Poincaré exponents. Finally, the vector $z(t)$ is the vector of modal variables. The three natural modes of a satellite include, in our numbering system, the variables z_1 and z_2, the free eccentricity and argument of perigee, which regresses at rate $\omega_1 = J_{11}$; the variables z_3, an in-track displacement paired with z_4, a change in orbital energy that will excite an in-track drift rate; and z_5, a change in orbital node paired with z_6, a change in orbital angular momentum that will excite a differential orbit plane regression. As in Ref. 3, there are two directions in which linear drift is possible, here forcing two off-diagonal unit entries in the Jordan form J. This solution will be referred to as the "free" motion of the satellite within a cluster.

Also in Ref. 2, this solution was used as the basis for perturbation theory to include all other perturbing forces in the problem. These are principally 1) second-order two-body and zonal harmonic terms, 2) sectoral and tesseral harmonics from the Earth's geopotential, and 3) air drag. It was found that these last two effects essentially perturb only the periodic orbit and are not important perturbers to the relative motion. Second-order two-body and zonal terms modify the mode shape matrix F, but do not introduce any new instabilities. All of these perturbations can be written as a forced solution to the time-periodic linear Floquet problem.

We turn our attention to the stationkeeping problem. That is, we will attempt to induce a satellite to follow a given free motion, in the presence of natural perturbations, as well as in the presence of discrepancies between an inertial frame truth-model numerical integration and the linearized solution. The approach will be to use impulsive maneuvers.

The choice of natural free motion trajectories as the reference solution is driven by two assumptions. First, we imagine that the satellites in the cluster will be very small vehicles and will not have substantial maneuvering capability. Furthermore, we assume that the cluster will need to operate for many years on orbit. These two assumptions mean that conservation of maneuvering fuel will be imperative. Whereas numerous authors have studied controlling the satellites in a cluster to follow nonnatural trajectories, the present author feels strongly that this approach will cost substantially more fuel and will drastically shorten the operational lifetime of the cluster. For example, it is argued that a satellite cluster designed to function as a phased array radar should have its individual satellites placed on trajectories that have the lowest possible control costs to ensure operational lifetime and that the choice of which natural trajectories are used should be made to optimize radar performance. Doing these steps in the reverse order will require controlling the satellite's motion to a reference solution that may optimize radar performance, but require much higher maneuver Δv costs.

We will shortly see that controlling the satellite's motion to conform as much as possible to the natural motion will offer maneuver costs that are dramatically low. The periodic orbit/Floquet theory of relative motion allows the characterization of these natural motions to a much higher accuracy than could be achieved heretofore. If the mission of the cluster can be achieved with the satellites following

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some subset of the free trajectories, then the approach of this paper offers substantial reduction in stationkeeping costs, with the attendant increase in mission lifetime. If the cluster mission absolutely mandates that the satellites fly trajectories that are not natural solutions to the dynamics, for example, an optical interferometer, then the approach of this paper will not be useful. Of course, if there is some overriding reason for flying a very nonnatural trajectory (as in the interferometer), then no sophisticated control work is necessary: The required control acceleration is just the difference between the acceleration in the required trajectory and the trajectory the satellite would naturally fly.

Two-Impulse Control

In general, with only the ability to maneuver a satellite impulsively, it will take two maneuvers to reposition a satellite from an initial state $z(t_0)$ to a given final state $z(t_3)$. We will almost always wish to specify final modal amplitudes $z_4 = 0$ and $z_6 = 0$ to prevent drift from the cluster reference orbit. (The one exception is initial deployment.) Imposing these two conditions places too many constraints on one maneuver, whereas they can be handled as a matter of course with two maneuvers. We will think of these maneuvers occurring at times t_1 and t_2, with the maneuver times imbedded within the overall time window as $t_0 \leq t_1 < t_2 \leq t_3$. The overall time interval (t_0, t_3) can be interpreted as an operational constraint, that is, an interval of time when the cluster will not be needed for its primary mission and orbit adjustments can be made. However, we shall also see that the introduction of bounding times t_0 and t_3 also has the effect of making the maneuver optimization time independent.

The modal state propagates with time as

$$z(t) = \exp[J(t - t_0)]\,z(t_0) \tag{2}$$

and the modal and orbital frame variables are related as

$$\boldsymbol{X}(t) = \begin{Bmatrix} \boldsymbol{r}(t) \\ \boldsymbol{v}(t) \end{Bmatrix} = F(t)\,z(t) \tag{3}$$

We recall from Wiesel[2] that the orbital frame velocity $\boldsymbol{v}$ is actually the inertial velocity of the spacecraft, but is resolved along the orbital frame axis system. At one level, this is an artifact of Hamiltonian dynamics, but it is very welcome here because maneuvers appear to change directly the inertial velocity. In an impulsive maneuver, then, where

$$\begin{aligned} \boldsymbol{r} &\rightarrow \boldsymbol{r} \\ \boldsymbol{v} &\rightarrow \boldsymbol{v} + \Delta\boldsymbol{v} \end{aligned} \tag{4}$$

the modal variables undergo the change

$$z \rightarrow z + F^{-1}(t) \begin{Bmatrix} 0 \\ \Delta\boldsymbol{v} \end{Bmatrix} \tag{5}$$

Thus, beginning at the initial modal state $z(t_0)$, we propagate forward to the first maneuver time t_1 and there perform the first maneuver. The modal state just after this maneuver is given by

$$z\left(t_1^+\right) = \exp[J(t_1 - t_0)]\,z(t_0) + F^{-1}(t_1) \begin{Bmatrix} 0 \\ \Delta\boldsymbol{v}_1 \end{Bmatrix} \tag{6}$$

Then propagating the preceding result to the time of the second maneuver t_2 and performing this maneuver gives

$$z\left(t_2^+\right) = \exp[J(t_2 - t_0)]\,z(t_0) + \exp[J(t_2 - t_1)]F^{-1}(t_1) \begin{Bmatrix} 0 \\ \Delta\boldsymbol{v}_1 \end{Bmatrix} + F^{-1}(t_2) \begin{Bmatrix} 0 \\ \Delta\boldsymbol{v}_2 \end{Bmatrix} \tag{7}$$

Finally, we propagate to the standard final time t_3, chosen to be the desired state at a time larger than the time interval we expect to search for optimal maneuvers. The modal state then is

$$z(t_3) = \exp[J(t_3 - t_0)]\,z(t_0) + \exp[J(t_3 - t_1)]F^{-1}(t_1) \begin{Bmatrix} 0 \\ \Delta\boldsymbol{v}_1 \end{Bmatrix} + \exp[J(t_3 - t_2)]F^{-1}(t_2) \begin{Bmatrix} 0 \\ \Delta\boldsymbol{v}_2 \end{Bmatrix} \tag{8}$$

This must equal our specified state at the final time.

Then with some manipulation, Eq. (8) can be put into the form

$$\Gamma(t_2, t_1)\Delta\boldsymbol{V} = \boldsymbol{b} \tag{9}$$

The vector on the right-hand side,

$$\boldsymbol{b} = z(t_3) - \exp[J(t_3 - t_0)]\,z(t_0) \tag{10}$$

does not depend on either maneuver time, but instead depends only on the difference between the chosen final state $z(t_3)$ and the initial conditions naturally propagated to the same final time t_3. The unknown vector

$$\Delta\boldsymbol{V} = \begin{Bmatrix} \Delta\boldsymbol{v}_1 \\ \Delta\boldsymbol{v}_2 \end{Bmatrix} \tag{11}$$

includes both impulsive maneuvers in one vector. All maneuver-time dependence appears in the matrix Γ, which has six rows and columns. Its first three columns are columns 4–6 of the matrix $\exp[J(t_3 - t_1)]F^{-1}(t_1)$, whereas the second three columns of Γ are columns 4–6 of the matrix $\exp[J(t_3 - t_2)]F^{-1}(t_2)$. Although this appears to depend on the final time t_3 also, in fact the modal variables are nearly constant on the timescale of one orbit, which is the major application that we have in mind. In this case, $\boldsymbol{b}$ will be a small vector. For initial deployment of the satellite cluster, it might be desirable to allow deployment to occur over several orbits. Also, notice immediately that Γ is singular when $t_2 - t_1$ is a multiple of the period τ of the orbit because the matrix $F(t)$ is periodic with this period.

The introduction of the final time t_3 is necessary. Some possible maneuvers, for example, adjusting a satellite's relative inclination, are both expensive and do not become less expensive with time. Other possible maneuvers, for example, adjusting the along-track displacement of a satellite, become less expensive the greater the time difference $t_2 - t_1$, and their cost theoretically goes to zero as $t_3 \rightarrow \infty$. This is unrealistic in the stationkeeping application, where the satellites must be returned to their nominal trajectories at infrequent intervals, although a larger value of t_3 might be desirable for initial deployment. In this work, we will choose $t_3 - t_0$ to be about one and one-half orbits, to not overconstrain the maneuver optimization, while at the same time to not allow the maneuvers to stretch over too long a period of time. After maneuvering, each satellite will have to determine its precise trajectory again, and depending on the cluster's mission, this may have to happen before normal operations can be fully resumed. Hence, we have elected to keep the interval over which maneuvers are to be performed relatively short.

Some attention must also be given to the choice of how the definition of "optimal" will be implemented. Our goal will be to minimize fuel usage in the long term. If the spacecraft is capable of full reorientation before each maneuver, then maneuvers are done with no geometric wastage. Then an appropriate cost function to minimize is

$$C_I = |\Delta\boldsymbol{v}_1| + |\Delta\boldsymbol{v}_2| \tag{12}$$

Alternately (and far more probably for a small spacecraft), the vehicle may not be free to completely reorient itself for maneuvers. If it is horizon stabilized in the orbital reference frame but still free to roll about the vertical axis, then horizontal and vertical components of the maneuver are done separately, and an appropriate cost function is

$$C_{II} = \sum_{i=1}^{2} \sqrt{\Delta v_{i1}^2 + \Delta v_{i2}^2} + \sqrt{\Delta v_{i3}^2} \tag{13}$$

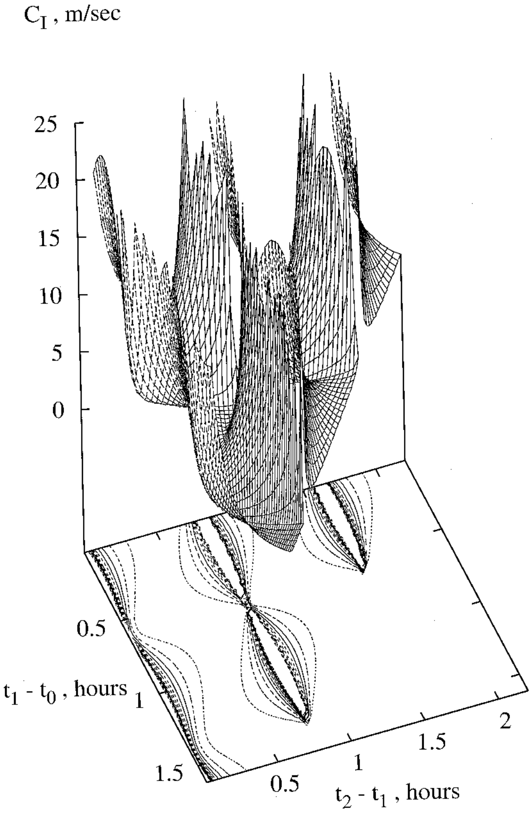

Fig. 1 Maneuver cost C_I for a large two-impulse transfer.

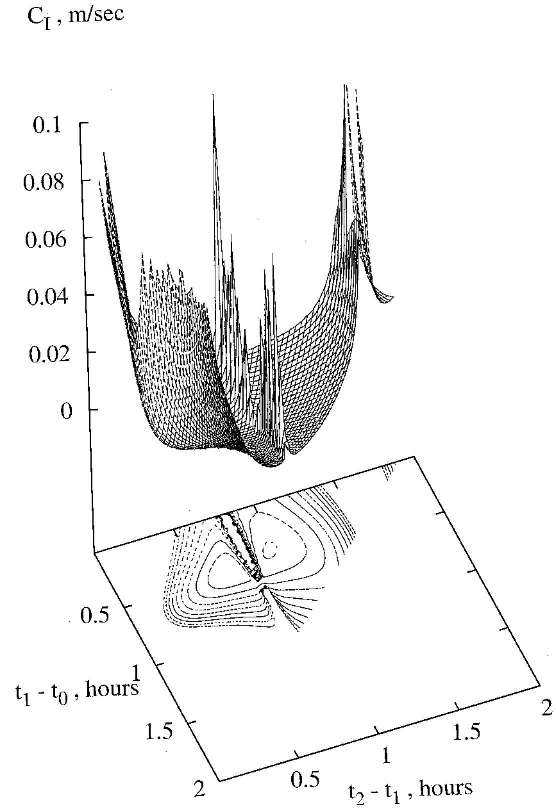

Fig. 2 Maneuver cost C_I for a typical stationkeeping two-impulse maneuver.

Finally, a fully horizon-stabilized satellite might also keep preferential axes aligned with the in-track and orbit normal directions and, therefore, would essentially do separate maneuvers in all three coordinate axes. Then an appropriate maneuver cost function will be

$$C_{III} = \sum_{i=1}^{2} \sum_{j=1}^{3} \sqrt{\Delta v_{ij}^2} \tag{14}$$

Figure 1 shows the cost function C_I for an initial deployment maneuver from $z = 0$ to nonzero values. About one orbital period is shown in both t_1 and $t_2 - t_1$ along the time axes. Large "ridges" appear in the cost near both one orbital period $\tau \approx 100$ min and at approximately $\tau/2$ in $t_2 - t_1$. The singularity near $\tau/2$ is also expected because the modal matrix $F(t)$ is periodic with one period, and so near $\tau/2$, some column vectors in F become nearly the negatives of their initial directions. The diagonal edge is due to the constraint that $t_2 \leq t_3$, which here was set slightly over one period. As we allow the final time t_3 to grow, some maneuvers, for example, in-track displacements z_3, will become cheaper, whereas orbit plane changes z_5 and free eccentricity/argument of perigee changes z_1 and z_2 will cost essentially the same amount. In fact, if the exponential matrix factors imbedded within $\Gamma(t_2, t_1)$ were constant, then Γ would be doubly periodic in both t_1 and $t_2 - t_1$, accounting for much of the appearance of Fig. 1. The optimal two-impulse burn to reposition a satellite within the cluster can then be found by using any reliable global minimization software on the system (9).

The first case shown is perhaps more typical of initial repositioning maneuvers that might be done to deploy a satellite cluster. The case shown in Fig. 2 is one that arose in the simulations to be reported next. Minimum maneuver cost in this case is less than 0.3 cm/s. Again prominent ridges near the orbital period and half the period appear. Also, although Fig. 2 shows two minima interior to the region searched, the actual minima may also appear at a corner or along the edge. The constraint boundary $t_2 > t_1$ is a perfectly legitimate constraint, but the choice of the end of the maneuver window final time t_3 we see as more in the nature of an operational constraint. Perhaps a satellite cluster whose principal mission is to observe Earth's land areas might use several consecutive orbits lying mostly over the oceans to perform the stationkeeping maneuvers, and this would dictate the stationkeeping time interval (t_0, t_3).

Numerical Experiments

We have conducted a series of numerical experiments simulating the functioning of the two-impulse control algorithm to stationkeep satellites to desired free oscillation modal states $z(t)$. The method is based on the separation of the complete orbital motion into a periodic orbit and its linearization solution via Floquet theory and forced solution to the linear system driven by all forces not included in the reference trajectory.

This is done with two simultaneous numerical integrations. The first integration is carried out in the Earth-centered inertial frame and is a direct, rectangular coordinate numerical integration of the orbit of the satellite, including all terms in the EGM96 model[4] through order and degree 14. Air drag is also included, with a ballistic coefficient $B^* = 0.1$ m^2/kg. The atmospheric density model was taken from Regan and Anandakrishnan.[5] It was noticed that their model produces air densities at 600-km altitude that are about an order of magnitude larger than that cited elsewhere,[6] and so the density has been reduced by that amount. Of course, an order of magnitude is well within the intrinsic variability of the air density at high altitude. This numerical integration is regarded as the truth model for the simulation.

We have used the same cluster orbit reported by Wiesel[2]: an orbital altitude of 0.1 Earth radii and an inclination of 1 rad. This avoids both the critical inclination and the artificial ease of stationkeeping satellite clusters in nearly polar orbits. Satellites have been placed randomly within the cluster, but with initial values $z_4 = z_6 = 0$, the two variables that will cause the cluster to disperse. Because we will shortly present results for a 10-satellite cluster, we will argue that the maneuver costs cited are statistical averages.

The theoretical solution includes the periodic orbit and modal matrix F, available as Fourier series, and the particular solution to this linear system including all of the perturbing forces included in the truth model. The second integration follows the forced solution for the perturbations to the periodic orbit modal variables,

$$\frac{d}{dt}z_{i,\text{forced}} = J_{i\alpha}z_{\alpha,\text{forced}} + F^{-1}(t)\mathcal{R}_Z^{(2)}\begin{Bmatrix} 0 \\ \boldsymbol{a}_p \end{Bmatrix} \tag{15}$$

The in-track mode z_3 has been suppressed in favor of the global periodic orbit phase Φ, as discussed by Wiesel.[2] We recall again that the forced solution is a perturbation of the periodic orbit itself because sectoral, tesseral, and air drag differential perturbations across the cluster are likely to be very small. Thus, there is only one forced solution, no matter how many satellites are in the cluster.

Then, the full numerically integrated inertial state from the truth model can be transformed to the nodal reference frame, the periodic orbit itself may be subtracted, and the result transformed into the modal variables. This process is the reverse of Eq. (1). Finally, subtracting the forced solution produces the free oscillation modal variables. The free oscillations are what we wish to control to a specified natural evolution. That is, the solution for the free variables will be controlled to follow

$$z_{\text{free}}(t) = \exp[J(t - t_0)]z_{\text{free}}(t_0) \tag{16}$$

the natural motion in the unperturbed model.

This does not exactly model all of the dynamics in the system. The second-order perturbations from the periodic orbit discussed by Wiesel[2] are ignored here because they are not included in the maneuver model of the preceding section. Calculating the free oscillation variables by subtracting the periodic orbit and forced perturbations from the inertial integration should also produce some errors due to loss of significant figures, as well as the normal truncation errors in numerical integration. The theoretical model is invariant to in-track displacements (the z_3 modal amplitude), and so we have made the choice that over our clusters the average in-track displacement $\bar{z}_3 = 0$. This is used to calculate a further (very small) correction to the orbital phase, which the author regards as a primitive orbit estimation method. It will keep the periodic orbit phase locked to the average in-track movement of the cluster itself. Second-order perturbations from air drag are also ignored.

Figure 3 shows the average maneuver requirement per satellite $\langle \Delta V \rangle$ for a cluster of 10 satellites maneuvering once per day over a period of a week. Error bars on the C_I curve give the 1-σ variation in the average maneuvers required. Initially the satellites in the cluster were displaced up to 2 km from the cluster center and were then controlled to follow their natural motion. All three cost functions are shown. The small amount of difference between the cost function C_{II}, which performs vertical maneuvers separately, and C_{III}, which performs separate maneuvers in all three directions, could be explained if most of the horizontal maneuvering is being performed in the in-track direction. If vertical and in-track maneuvers dominate and maneuvers in the orbit normal direction are very small, perhaps a not unexpected situation, then this would explain the performance benefit offered by a satellite that can fully orient itself, combining the vertical and in-track maneuvers into one. It would also explain the lack of improvement offered by retaining roll orientation about the vertical axis.

The reason that control costs are growing with time, for any of the cost functions, is that the numerical integration of the individual orbits and the periodic orbit/Floquet model are running independently of each other. The numerical integration and the theoretical model start with the same initial conditions, and the phase angle of the periodic orbit is updated to track the centroid of the cluster itself, but no other attempt is made to actually fit the model to the integration. Most of the maneuvers have to do with maintaining the in-track spacing of the satellites themselves. Figure 4 shows the in-track modal coordinates z_3 for a 10-satellite cluster. Because all satellites were started with the conjugate energy change mode $z_4 = 0$, the in-track displacement z_3 should be constant. Maneuvers are repeatedly made to return each satellite successfully to the specified in-track coordinate value. It then immediately begins to depart again. Note that the drift direction is different for different satellites within the cluster. The cluster, as a whole, is attempting to disperse. This behavior also appears when air drag is excluded from the model, indicating that differential air drag is not the cause of the dispersal.

This behavior not only remains when air drag is removed from the dynamics, it even remains when the dynamic model is reduced to just the two-body and zonal harmonic terms in which the periodic orbit/Floquet theory is constructed. This indicates that the difficulties with in-track maneuvers is due to the neglected second-order secular terms in the baseline model. Wiesel[2] found that, although second-order zonal perturbations do not introduce any additional instabilities of the cluster, the second-order terms can modify the secular rates expected from the linear Floquet theory. To check this, the initial size of the satellite cluster reported here (about 4 km

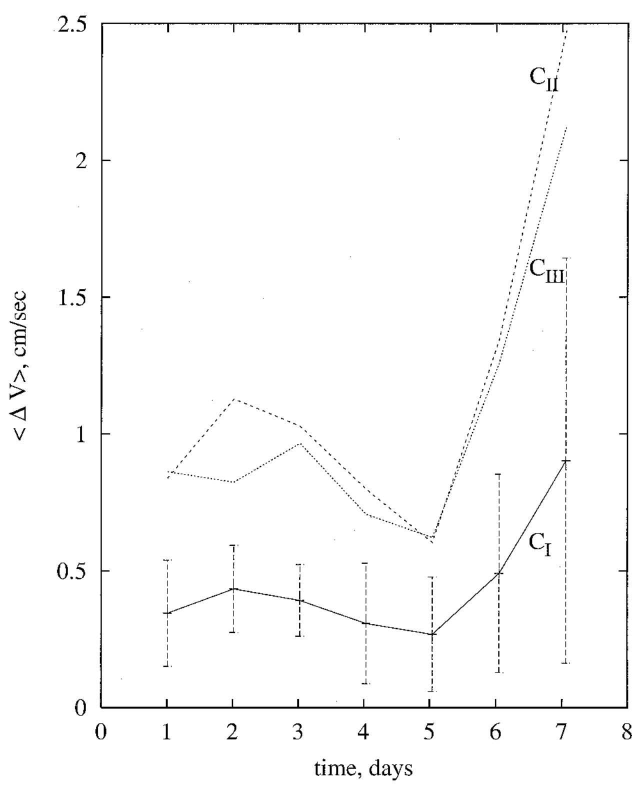

Fig. 3 One week of stationkeeping for a 10-satellite cluster with cost functions C_I, C_{II}, and C_{III}; performing daily maneuvers.

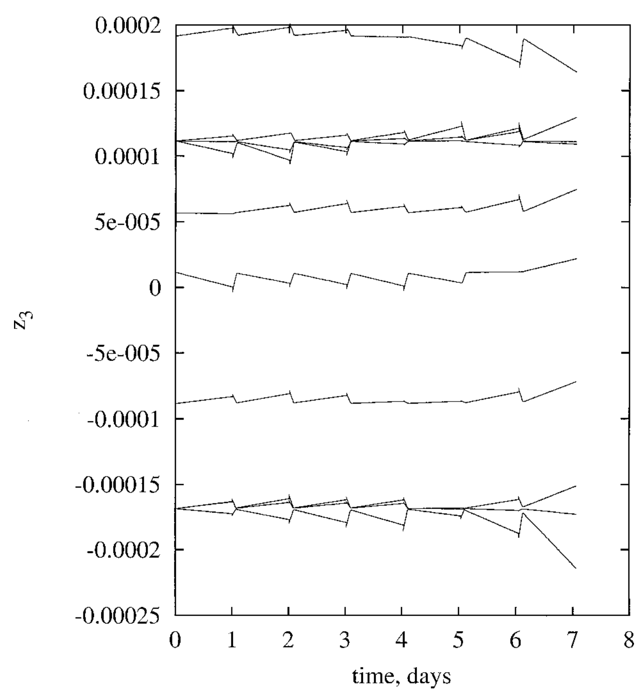

Fig. 4 In-track modal coordinates z_3 for a 10-satellite cluster; maneuvers appear as a sawtooth pattern once per day.

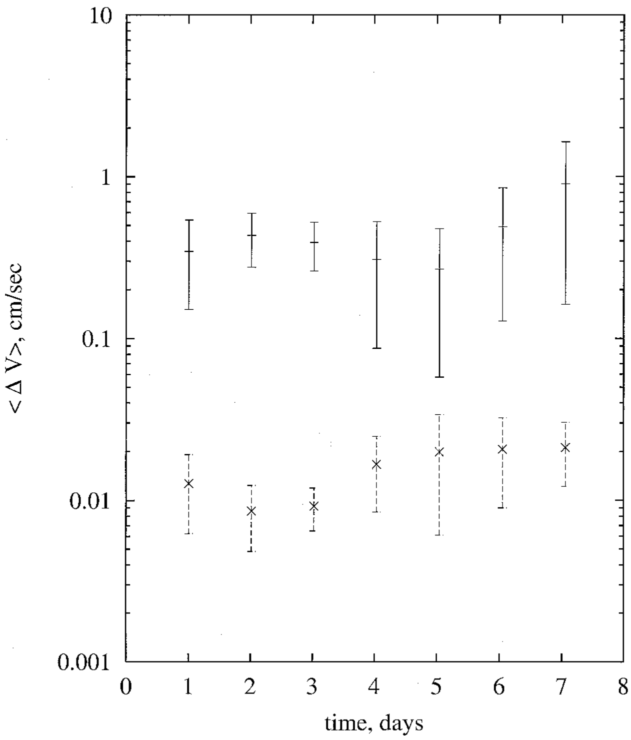

Fig. 5 C_I **maneuver costs for a 4-km cluster (top) and a 400-m cluster (bottom).**

overall) was reduced by an order of magnitude. Average daily maneuvers were reduced from the value of $\frac{1}{2}$ cm/s per day to a value close to 0.1 mm/s per day, or a reduction of a factor of 50. The C_I cost function for this case is shown in Fig. 5, along with the corresponding result from the earlier case. Again, error bars indicate the 1-σ variation of the average.

In the performance of this work, the question of the level at which the numerical integrations can be trusted was repeatedly asked. This question is paramount because maneuvers could be forced either by inaccuracies in the theoretical model or by inaccuracies in the numerical integration of the truth model. There is an unavoidable and severe loss of numerical accuracy when the inertial frame integrations are converted to modal variables, due to the subtraction of the periodic orbit coordinate values from the numerically integrated coordinates. The reduction of average maneuver requirements by almost two orders of magnitude when the cluster extent was reduced by one order of magnitude indicates that the numerical integrations are trustworthy. However, if second-order terms in the in-track dynamics are the cause of the maneuvers observed with the larger cluster size, a reduction in maneuvering by two orders of magnitude is exactly what would have been expected when the cluster size is reduced by a factor of 10, and the observed improvement of a factor of 50 meets most of this expectation. Because we are worried about deviations on the order of tens of meters over a week, the entire process is running against the limits of numerical accuracy in double-precision computer arithmetic. The author is convinced that second-order in-track dynamics are driving most of the maneuver requirements reported in this paper. Including these effects should reduce the maneuver requirements reported here even further.

Conclusions

We have constructed an optimal first-order maneuver theory for stationkeeping satellites in a cluster. The individual satellites are controlled to follow their own natural motion, after perturbations of the cluster as a whole are subtracted. Numerical simulations have been performed, where the periodic orbit/Floquet model is run open loop against an inertial frame numerical integration, incorporating a full geopotential model and air drag. Simulations show average stationkeeping maneuver costs of approximately 0.5 cm/s per day for a cluster with a 4-km size, to less than 0.1 mm/s per day for a cluster with a size of 400 m. Either number may be conservative because there are good indications that the in-track dynamics model can be improved. This would need to occur both in the propagation model, as in Ref 2, and also in the maneuver model. Maneuvers can probably be minimized by inserting nonzero values of the modal components z_4 and z_6. In the linear model, this will simply produce additional secular drift rates. In the nonlinear model, they might be used to counter the effects of the second-order secular terms. We note that it is extremely common in classical perturbation theory to need to carry the perturbation analysis of the in-track motion to one order higher than the rest of the analysis. The same is apparently true in control of satellite clusters. This remains an area under investigation.

In either case, the obvious next step is to replace the open-loop theoretical model with an estimation algorithm. A successful estimation algorithm will, of course, keep the theoretical model locked to the numerical integration (or eventually, locked to reality). It will also introduce another source of modeling error, the degree to which the estimate and reality are different. This will add to the maneuver costs because at least part of each maneuver will be in response to the error in the estimate. It is hoped that this will mitigate the maneuver cost growth shown in the work reported here.

Acknowledgments

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